Calculate Liquid Volumes in Tanks with Dished Heads

A downloadable spreadsheet simplifies the use of these equations

Daniel R. Crookston, Champion Technologies

Reid B. Crookston, Retired

This article presents equations that allow the user to calculate liquid volume as a function of liquid depth, in both vertically and horizontally oriented tanks with dished heads. The equations accommodate all tank heads that can be described by two radii of curvature (torispherical heads). Examples include: ASME flanged & dished (F&D) heads, ASME 80/10 F&D heads, ASME 80/6 F&D heads, standard F&D heads, shallow F&D heads, 2:1 elliptical heads and spherical heads. Horizontal tanks with true elliptical heads of any aspect ratio can also be accommodated using this methodology.

This approach can be used to prepare a lookup table for a specific tank, which yields liquid volumes (and weights) for a range of liquid depths. The equations can also be applied directly to calculate the liquid volume for a measured liquid depth in a specific tank. Such calculations can be executed using a spreadsheet program, a programmable calculator or a computer program. Spreadsheets that perform these calculations are available from this magazine (search for this article online at www.che.com, and see the Web Extras tab).

Problem background

Tanks with dished heads are found throughout the chemical process industries (CPI), in both storage and reactor applications. In some cases, liquid volume calibrations of these vessels exist, but for many, the liquid volumes must be calculated. Traditional methods of calculation can be cumbersome, and some lack precision or offer little or no equation derivation.

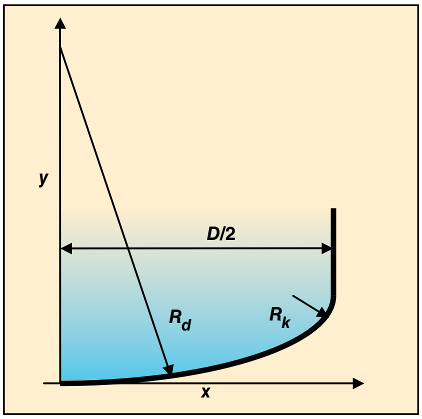

FIGURE 1. This figure shows the relevant radii of curvature and the coordinate system used for a vertical tank

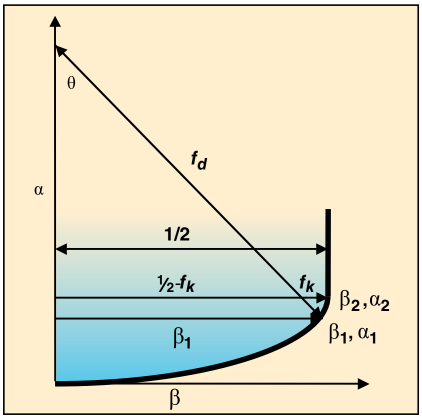

FIGURE 2. This two-dimensional view of the tank head is shown using dimensionless parameters

The equations presented below are mathematically precise and have a detailed derivation. The spreadsheets that are offered to perform the calculations produce a table of liquid volumes for a range of liquid depths that are suitable for plant use. This table is generated by entering four parameters that define key tank dimensions. An operator could use such a spreadsheet table in lookup mode, using interpolation if necessary. One could also turn the tabular values into a plot.

Each spreadsheet also has a calculator function, which requires the user to enter only the tank geometry parameters and liquid depth and the spreadsheet quickly returns the liquid volume. The spreadsheets can be used with handheld devices (such as a Blackberry or iPhone) that can run an Excel spreadsheet. For certain applications, one may want to show only the calculator function for a given vessel, so that an operator would only need to enter a liquid level to quickly calculate the corresponding liquid volume.

A number of tank heads have a dished shape, and the equation development discussed below handles all of those where the heads can be described by two radii of curvature.

Doolittle [*1*] presents a graphical representation of liquid volumes in both horizontal and vertical tanks with spherical heads. The calculation of the liquid in the heads is approximate. The graph shows lines for tank diameters from 4 to 10 ft, and tank lengths from 1 to 50 ft. The accuracy of the liquid volume depends on certain approximations and the precision of interpolations that may be required.

Perry [*2*] states that the calculation of volume of a partially filled tank "may be complicated." Tables are given for horizontal tanks based on the approximate formulas developed by Doolittle.

Jones [*3*] presents equations to calculate fluid volumes in vertical and horizontal tanks for a variety of head styles. Unfortunately, no derivation of those equations is offered. As of the time of this writing, there were no Internet advertisements offering spreadsheets to solve the equations. Meanwhile, without adequate equation derivations, one would be unsure what one is solving, and thus, the results would be suspect.

By contrast, this article provides

exact equations for the total volume of the heads and exact equations for liquid volumes, for any liquid depth for any vertical or horizontal tank with dished heads. The popular 2:1 elliptical heads are actually fabricated as approximate shapes by using variations of the two-radii designs.

In addition, this article also presents the exact equations for true elliptical heads of any ratio (not limited to 2:1). Provided below are descriptions of the equation development, guidance on how to use the spreadsheets, and a discussion of a sample application for both a vertical and a horizontal tank.

Types of dished tank heads

Figure 1 shows the relevant radii of curvature and the coordinate system used. All symbols are defined in the Nomenclature Section on p. 59. It is convenient to present the equation development in terms of dimensionless variables. By normalizing all lengths by the tank diameter, the diameter is absent from all equations expressed in the dimensionless coordinates. The two radii (dish radius and knuckle radius) that describe the dished heads can be expressed as follows:

$$R_d = f_d D \tag{1}$$

$$R_k = f_k D \tag{2}$$

Table 1 presents standard dished tank heads that are described by this work.

Radius as a function of depth

For convenience, the derivation in this section describes a tank with vertical orientation. However, the derivation applies to horizontal tanks as well. The equations are used in the integrations described in the subsequent two sections, which yield the liquid volumes for vertical and horizontal tanks.

For the dished heads considered here, two radii define the shape. The bottom region of the head is spherical and has a radius that is proportional to the diameter of the cylindrical region of the tank (see Equation 1). This is referred to as Region 1.

Above that is Region 2, which is called the knuckle region. Its radius of curvature is shown in Figure 1. It can also be normalized by the tank diameter (see Equation 2).

The last concept needed to define the dish shape is that the curvatures of the two radii are equal at the plane where Regions 1 and 2 join. That will be explained further in the equation development that follows.

The coordinate system for the equations is shown in Figure 1. The origin of the coordinate system is chosen to be at the bottom-most point in the tank. For Region 1, the equation for the tank radius, x, in terms of the height, y, is as follows:

$$x^2 + \left(f_d D - y\right)^2 = f_d^2 D^2 \tag{3}$$

This equation can be expressed via the following dimensionless variables:

$$\alpha = y / D \tag{4}$$

$$\beta = x / D \tag{5}$$

Substituting Equations 4 and 5 into Equation 3 gives the final dimensionless equation for Region 1, as shown in Equation 6:

$$\beta^2 + \left(f_d - \alpha\right)^2 = f_d^2 \tag{6}$$

For Region 2, the equation for the tank radius, x, in terms of the height, y, is:

$$\left(x - x_k\right)^2 + \left(y - y_k\right)^2 = f_k^2 D^2 \tag{7}$$

Where (x_k, y_k) is the coordinate location of the center of the knuckle radius. By substituting Equations 4 and 5, Equation 7 is made dimensionless, as shown in Equation 8:

$$\left(\beta - \beta_k\right)^2 + \left(\alpha - \alpha_k\right)^2 = f_k^2 \tag{8}$$

The x-coordinate of the knuckle radius, x_k, must be:

$$x_k = \frac{D}{2} - f_k D \tag{9}$$

Equation 9 can be made dimensionless, as shown in Equation 10:

$$\beta_k = 0.5 - f_k \tag{10}$$

Making that substitution into Equation 8 gives the final dimensionless equation for Region 2:

$$\left(\beta - 0.5 + f_k\right)^2 + \left(\alpha - \alpha_k\right)^2 = f_k^2 \tag{11}$$

TABLE 1. STANDARD DISHED TANK-HEAD TYPES

Tank head style	Dish radius factor, f_d	Knuckle radius factor, f_k
ASME flanged & dished (F&D)	1.000	0.060
ASME 80/10 F&D	0.800	0.100
ASME 80/6 F&D	0.800	0.060
2:1 Elliptical	0.875	0.170
Spherical	0.500	0.500
Standard F&D	1.000	2 in./D
Shallow F&D	1.500	2 in./D

Region 3 is the cylindrical portion of the tank with a constant diameter, with β equaling 0.5.

Next, one must determine the coordinates of the point where the curves for Regions 1 and 2 come together. Working with the dimensionless variables, β and α, and using the subscript 1 to denote the top of Region 1, we seek to find α_1 (the dimensionless coordinate of the top of Region 1), such that Equations 6 and 11 both give the same value for α_1 (given the same value of β_1), and such that the curvature is continuous at the intersection.

Figure 2 is a two-dimensional view of the tank head using dimensionless parameters. The radius of the spherical region is drawn through the origin of the knuckle radius. The point where that line intersects the head identifies where Regions 1 and 2 join. At that point, the curvatures of the spherical region and the knuckle region are identical. The angle between the radius of that spherical region and the tank center line is denoted as θ. We can write the follow three trigonometric expressions involving that angle:

$$\sin\theta = \frac{1/2 - f_k}{f_d - f_k} \tag{12}$$

$$\sin\theta = \frac{\beta_1}{f_d} \tag{13}$$

$$\cos\theta = \frac{f_d - \alpha_1}{f_d} \tag{14}$$

Recognizing the following trigonometric identity

$$\sin^2\theta + \cos^2\theta = 1 \tag{15}$$

We substitute Equations 12 and 14 into Equation 15 and solve for α_1:

$$\alpha_1 = f_d \left[1 - \sqrt{1 - \left(\frac{1/2 - f_k}{f_d - f_k}\right)^2}\right] \tag{16}$$

The value of β_1 can be calculated by combining Equations 12 and 13:

TABLE 2. DEFINED AND CALCULATED PARAMETERS FOR DISHED TANK HEADS

Tank head style	f_d	f_k	α_1	β_1	α_2	β_2
ASME F&D	1.000	0.06	0.1163166103	0.4680851064	0.1693376137	0.5
ASME 80/10 F&D	0.800	0.10	0.1434785547	0.4571428571	0.2255437353	0.5
ASME 80/6 F&D	0.800	0.06	0.1567794689	0.4756756757	0.2050210088	0.5
2:1 Elliptical	0.875	0.17	0.1017770340	0.4095744681	0.2520032103	0.5
Spherical	0.500	0.50	0.5000000000	0.5000000000	0.5000000000	0.5

TABLE 3. RATIO OF TOTAL HEAD CAPACITY TO D^3 FOR VARIOUS DISHED HEADS

Tank head style	f_d	f_k	α_1	$\alpha_2 = \alpha_k$	C
ASME F&D	1.000	0.06	0.116317	0.169338	0.0809990
ASME 80/10 F&D	0.800	0.10	0.143479	0.225544	0.1098840
ASME 80/6 F&D	0.800	0.06	0.156779	0.205021	0.0945365
2:1 Elliptical	0.875	0.17	0.101777	0.252003	0.1337164
Spherical	0.500	0.50	0.500000	0.500000	0.2617994

$$\beta_1 = f_d\left(\frac{1/2 - f_k}{f_d - f_k}\right) \quad (17)$$

To calculate α_2 we apply the Pythagorean Theorem to the right triangle whose hypotenuse is a line between the origin of the spherical radius and the origin of knuckle radius, as shown in Equation 18:

$$\left(1/2 - f_k\right)^2 + \left(f_d - \alpha_2\right)^2 = \left(f_d - f_k\right)^2 \quad (18)$$

Solving that for α_2 gives:

$$\alpha_2 = f_d - \sqrt{f_d^2 - 2f_d f_k + f_k - 1/4} \quad (19)$$

α_k is located at the top of Region 2, so

$$\alpha_2 = \alpha_k \quad (20)$$

At the top of Region 2, the head radius equals the radius of the cylindrical portion, so β_2 equals ½.

For Region 3, the radius is constant and is simply half the tank diameter. So, the expression for the tank radius is shown in Equation 21:

$$\beta = 0.5 \quad \text{for} \quad \alpha_2 \le \alpha \le \alpha_3 \quad (21)$$

It is not necessary to construct equations for β as a function of α in Regions 4 and 5. For vertical tanks, the volumes for liquid levels in those regions can be calculated from the equations for Region 1 and 2 (presented below). For horizontal tanks, the liquid volume in the right-hand head equals that of the left-hand head for the symmetrical tanks discussed here.

The value for α_1 (top of Region 1) for each head style was determined by solving Equation 16. β_1 is given by Equation 17. α_2 is equivalent to α_k, and its value is given by Equation 19. At the top of the tank, α_5 is the tank height, H, divided by the diameter, or

$$\alpha_5 = H / D \quad (22)$$

Since the two heads are taken to be the same shape:

$$\alpha_4 = \alpha_5 - \alpha_1 \quad (23)$$

$$\alpha_3 = \alpha_5 - \alpha_2 \quad (24)$$

So, the values of α_1 through α_5 are thusly constructed.

Values for α_1, β_1, α_2 and β_2 for the various tank head styles considered here are summarized in Table 2.

One should recognize that the parameters in Table 2 apply to all of the torispherical tank head styles, regardless of the tank diameter. That is one of the benefits of working with the dimensionless parameters.

One use for the α_2 values would be to calculate the distance from the end of a dished head to the plane through the boundary between Regions 2 and 3. So, for example, if one had ASME flanged and dished (F&D) heads of a tank with a 100-in. I.D. for which α_2 equals 0.1693376137, that length would be 0.1693376137 times 100 in., or 16.934 in.

The last two tank head styles listed in Table 1 (standard flanged & dished, and shallow flanged & dished) require a somewhat different treatment, since the radius of curvature for the knuckle region in each case is a fixed 2 in. rather than a fixed fraction of the tank diameter. While all the equations above still apply, one must determine the α and β parameters in Table 2 for each individual tank.

So, for example, if one had standard flanged & dished heads on a 100 in. dia. tank, f_k would be 0.02 and f_d would be 1.0. Those values would be used in Equation 16 to find α_1. One would, in turn, use the appropriate equations to calculate β_1, α_2, and β_2. All the equations in the following sections for the tank volume and liquid volume also apply.

Liquid volume as a function of depth for vertical tanks

Liquid volume in Region 1. The liquid volume, v_i, in any tank region i is simply

$$v_i = \int_{y_{i-1}}^{y} \pi x^2 dy \quad (25)$$

Replacing x and y by their dimensionless expressions in Equations 4 and 5 gives

$$v_i = \pi D^3 \int_{\alpha_{i-1}}^{\alpha} \beta^2 d\alpha \quad (26)$$

For Region 1, substituting for β^2 from Equation 6 and integrating gives

$$v_1 = \pi D^3\left[f_d \alpha^2 - \alpha^3/3\right] \quad \text{for} \quad 0 \le \alpha \le \alpha_1 \quad (27)$$

The total capacity of Region 1, denoted as V_1, can be calculated by putting α_1 and a value for D into Equation 27. This will also be the total tank capacity of Region 5, denoted as V_5.

Liquid volume in Region 2. For Region 2, the liquid volume is calculated using Equation 28:

$$v_2 = \pi D^3 \int_{\alpha_1}^{\alpha} \beta^2 d\alpha \quad (28)$$

Substituting for β from Equation 11 and integrating gives Equation 29:

Equation 29: (see box on p. 56) (29)

As discussed above, α_k is identical to α_2 (see Equation 20), so that substitution could be made in Equation 29.

The total capacity of Region 2, denoted as V_2, can be calculated by putting α_2 in place of α in Equation 29. This will also be the total tank capacity of Region 4, denoted as V_4.

Liquid volume in Region 3. Carrying out the integration in Equation 26 for Region 3 with the substitution from Equation 21 yields the liquid volume in Region 3, as shown next:

$$v_3 = \frac{\pi D^3}{4}\left[\alpha - \alpha_2\right] \quad \text{for} \quad \alpha_2 \le \alpha \le \alpha_3 \quad (30)$$

The total capacity of Region 3, denoted

EQUATIONS 29, 31, 36

$$v_2 = \pi D^3 \left\{ \left[(0.5-f_k)^2 + f_k^2 \right](\alpha-\alpha_1) - \frac{1}{3}\left[(\alpha-\alpha_k)^3 - (\alpha_1-\alpha_k)^3 \right] + (0.5-f_k)\left[(\alpha-\alpha_k)\sqrt{f_k^2-(\alpha-\alpha_k)^2} - (\alpha_1-\alpha_k)\sqrt{f_k^2-(\alpha_1-\alpha_k)^2} + f_k^2 \sin^{-1}\frac{(\alpha-\alpha_k)}{f_k} - f_k^2 \sin^{-1}\frac{(\alpha_1-\alpha_k)}{f_k} \right] \right\} \text{ for } \alpha_1 \le \alpha \le \alpha_2 \quad (29)$$

$$v_4 = V_4 - \pi D^3 \left\{ \left[(0.5-f_k)^2 + f_k^2 \right](\alpha_5-\alpha-\alpha_1) - \frac{1}{3}\left[(\alpha_5-\alpha-\alpha_k)^3 - (\alpha_1-\alpha_k)^3 \right] + (0.5-f_k)\left[(\alpha_5-\alpha-\alpha_k)\sqrt{f_k^2-(\alpha_5-\alpha-\alpha_k)^2} - (\alpha_1-\alpha_k)\sqrt{f_k^2-(\alpha_1-\alpha_k)^2} + f_k^2 \sin^{-1}\frac{(\alpha_5-\alpha-\alpha_k)}{f_d} - f_k^2 \sin^{-1}\frac{(\alpha_1-\alpha_k)}{f_d} \right] \right\} \text{ for } \alpha_3 \le \alpha \le \alpha_4 \quad (31)$$

$$C = \pi\left[f_d\alpha_1^2 - \alpha_1^3/3 \right] + \pi\left\{ \left[(0.5-f_k)^2 + f_k^2 \right](\alpha_2-\alpha_1) + \frac{1}{3}(\alpha_1-\alpha_k)^3 + (0.5-f_k)\left[(\alpha_k-\alpha_1)\sqrt{f_k^2-(\alpha_1-\alpha_k)^2} - f_k^2 \sin^{-1}\frac{(\alpha_1-\alpha_k)}{f_k} \right] \right\} \quad (36)$$

as V_3, can be calculated by putting α_3 into Equation 30 in place of α.

Liquid volume in Region 4. If the liquid level is in Region 4, the volume can be determined from the volume equation for Region 2, Equation 29. For a liquid level α in Region 4, the height of the tank's vapor space would be $(\alpha_5 - \alpha)$. The volume of the vapor space in Region 4 would be equivalent to the liquid volume in Region 2 if the level were at a depth of $(\alpha_5 - \alpha)$. So, to calculate the liquid volume in Region 4, we take the capacity of Region 4 (equivalent to the capacity of Region 2) and subtract the vapor space in Region 4.

Equation 31: (see box above) **(31)**

Liquid volume in Region 5. In an analogous manner, the liquid volume in Region 5 is:

$$v_5 = V_5 - \pi D^3 \left[f_d(\alpha_5-\alpha)^2 - (\alpha_5-\alpha)^3/3 \right] \text{ for } \alpha_4 \le \alpha \le \alpha_5 \quad (32)$$

Tank capacity and total liquid volume. The total tank capacity is

$$V_T = 2V_1 + 2V_2 + V_3 \quad (33)$$

The final expression for the liquid volume is shown in Equation 34:

$$v = \sum_{i=1}^{i-1} V_i + v_i \quad (34)$$

Where the v_i and V_i terms are given by Equations 27, 29, 30, 31, and 32 for the five regions.

Capacities of dished heads. The total head volume (capacity) for each dished head considered in this article can be calculated by adding the volumes of Region 1 (Equation 27 with $\alpha = \alpha_1$) and Region 2 (Equation 29 with $\alpha = \alpha_2$). One can see the result will be an equation of this form:

$$V_h = CD^3 \quad (35)$$

Where C is calculated as:

Equation 36: (see box above) **(36)**

Table 3 shows the value of C for each type of head considered here.

Perry [2] gives an approximate value for C for an ASME F&D head as 0.0809, which is quite close to the precise value given in Table 3.

Liquid volume as a function of depth for horizontal tanks

The liquid depth, d, in a horizontal tank is measured in the cylindrical region. Calculation of the liquid volume in the cylindrical region of the tank is straightforward; calculating the liquid volume in the two dished heads is more challenging. First, one needs to recognize that every possible tank cross-section formed by planes perpendicular to the tank's center axis will be a circle. In the dished regions, if there is liquid at any given plane, the area of that liquid A_L will be what is termed a segment of the circular cross-section. One can calculate the liquid volume between any two cross-sectional planes by integrating the following:

$$v_L = \int_{y_a}^{y_b} A_L dy \quad (37)$$

The coordinate system for horizontal tanks is shown in Figure 3. We begin the development of the liquid volume equation by looking at the cylindrical region, and follow that by dealing with the dished regions.

Liquid volume in the cylindrical region. If one envisions a cross-section perpendicular to the tank axis in the cylindrical region of a horizontal tank with a liquid depth d, the area of a segment representing the liquid would have an area of

$$A_{LC} = 2\int_{-R}^{-(R-d)} x dy \quad (38)$$

where the center of the coordinate system is the tank's centerline in a plane perpendicular to that centerline, and R is the tank radius. The equation for the circle formed by the intersection of the tank with that plane is shown in Equation 39:

$$x^2 + y^2 = R^2 \quad (39)$$

Substituting Equation 39 into 38 and integrating gives:

$$A_{LC} = (d-R)\sqrt{2dR-d^2} + R^2\sin^{-1}\frac{d-R}{R} + \frac{\pi R^2}{2} \quad (40)$$

Defining a dimensionless liquid depth

$$\delta = d/D \quad (41)$$

and substituting Equation 41 into Equation 40, and replacing R with $D/2$ gives

$$A_{LC} = D^2\left[(\delta-\frac{1}{2})\sqrt{\delta-\delta^2} + \frac{1}{4}\sin^{-1}(2\delta-1) + \frac{\pi}{8} \right] \quad (42)$$

Given that the length of the cylindrical region is $(\alpha_3 - \alpha_2) D$, the volume of liquid in the cylindrical region is just area times length, or

$$v_{LC} = (\alpha_3-\alpha_2)D^3 \left[(\delta-\frac{1}{2})\sqrt{\delta-\delta^2} + \frac{1}{4}\sin^{-1}(2\delta-1) + \frac{\pi}{8} \right] \quad (43)$$

Liquid volume in the tank heads. The liquid volume in the dished regions is arrived at by analogous reasoning to that used for the cylindrical region. Again, planes constructed perpendicular to the tank axis will intersect the dished head giving circular shapes.

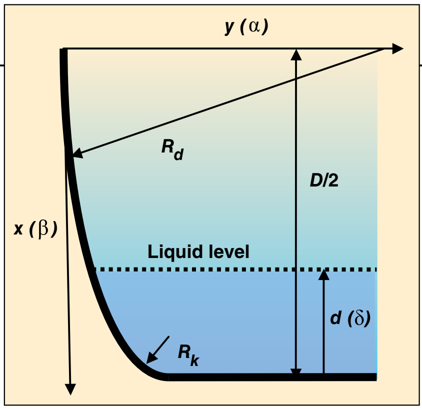

FIGURE 3. The coordinate system for a horizontal tank is shown here

EQUATION 48

$$A_{LD} = D^2\left[(\delta-\frac{1}{2})\sqrt{-\delta^2+\delta+\beta^2-\frac{1}{4}}+\beta^2\sin^{-1}\frac{\delta-\frac{1}{2}}{\beta}+\frac{\pi\beta^2}{2}\right]$$

$$\text{where } \left(\frac{1}{2}-\beta\right)\le\delta\le\left(\frac{1}{2}+\beta\right) \quad (48)$$

The radii of those circles will depend on the curvature of the dish and, as such, will vary with α, the dimensionless distance from the left-hand end of the tank. Also, for a given liquid depth in the cylindrical region, the liquid depth at a cross-section in the dished head will be less than in the cylindrical region because of the dish curvatures.

Referring to Figure 4, a schematic view looking toward the left-hand tank dished head, the outer circle represents the cylindrical diameter and the inner circle represents a cross-section in the dished region. The horizontal dashed line represents a liquid level, shown here in the lower half of the tank. The radius of the dished head at the cross-section is x, or β in the dimensionless coordinates, and the liquid height at the cross section is h. We can normalize that liquid depth by defining a dimensionless variable, γ, as shown:

$$\gamma = h / D \quad (44)$$

We relate h, d and x as follows:

$$= d - \left(\frac{D}{2}-x\right)$$

$$\text{here } \left(\frac{D}{2}-x\right)\le d\le\left(\frac{D}{2}+x\right) \quad (45)$$

In other words, if the liquid depth is below (D/2 – x), there is no liquid area at the cross-section, and if the depth is above (D/2 + x), then the entire circular area is covered. Equation 45 can be written in terms of the dimensionless variables

$$\gamma = \delta - \frac{1}{2} + \beta$$

$$\text{where } \left(\frac{1}{2}-\beta\right)\le\delta\le\left(\frac{1}{2}+\beta\right) \quad (46)$$

We can write an equation for the liquid area of a cross-section in the dished region (perpendicular to the main axis) by analogy to Equation 40, where the radius, x, replaces R, and where the liquid depth, h, replaces d.

$$A_{LD} = (h-x)\sqrt{2hx-h^2} + x^2\sin^{-1}\frac{h-x}{x}+\frac{\pi x^2}{2} \quad (47)$$

Next, we convert to dimensionless variables and substitute from Equation 46 to create Equation 48:

Equation 48: (see box, above) (**48**)

To get the liquid volume in the two dished tank heads, apply Equation 37:

$$v_{LD} = 2D\left[\int_0^{\alpha_1} A_{L1}d\alpha + \int_{\alpha_1}^{\alpha_2} A_{L2}d\alpha\right] \quad (49)$$

If we were able to perform this integration and get a closed-form solution, we would substitute Equation 48 for A_{L1}, substitute for β in Region 1 from Equation 6 and perform similar substitutions for Region 2. That would give two integrals, each only involving the variable α. While it is not possible to perform those integrations analytically, it is possible to perform the integrations numerically.

We use Simpson's Rule for the numerical integration. It is based on having an odd number of equally spaced intervals in the independent variable, in this case α, and calculating the corresponding values for the areas. We chose to use 100 intervals between α = 0 and α = α_2. The numerical integration was performed as part of a spreadsheet, described below in the Results section. Simpson's Rule for any three consecutive integration points is

$$v_{LD} = \frac{2D(\Delta\alpha)}{3}\left(A_{La}+4A_{Lb}+A_{Lc}\right) \quad (50)$$

Where Δα is α_2/100 and A_{La}, A_{Lb}, and A_{Lc} are the areas at the three corresponding α points. The liquid volume in the two heads is calculated by applying Simpson's Rule to each of the three cross-sections, summing the parts to cover the 101 cross-sections, and doubling that to account for the two heads. The total liquid in the tank is the sum of the liquid in the cylindrical region and the two heads.

Liquid volume in true elliptical tank heads. Elliptical heads are commonly used on horizontal tanks. While a true ellipse doesn't conform to the definition of heads characterized by two radii of curvature, their shape is much simpler, and the contained liquid volume can be calculated by simple algebraic formula, derived below.

For this exercise we imagine an orthogonal x-y-z coordinate system with its origin at the center of an ellipsoid formed by revolving an ellipse about the z-axis. The z-axis is taken to coincide with the centerline of the cylindrical portion of the tank, and the x and y-axes are in the plane perpendicular to the z-axis at the center of the ellipsoid, with the y-axis being vertical. The equation that describes the surface of the ellipsoid is:

$$\frac{x^2}{R^2}+\frac{y^2}{R^2}+\frac{z^2}{Z^2}=1 \quad (51)$$

So, the x-axis intersects the ellipsoid at R, the y-axis intercepts at R, and the z-axis intercepts at Z. As an example, if one had a true 2:1 elliptical head, Z would equal R/2. We define e, the ratio of the intercepts of the ellipsoid, such that

$$Z = \frac{R}{e} = \frac{D}{2e} \quad (52)$$

Straightforward integration shows that the area of an ellipse represented by Equation 53:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (53)$$

is Equation 54 [*4*]:

$$A = \pi ab \quad (54)$$

With the coordinate system described above for an ellipsoid, the y-axis will be the vertical axis, and the liquid surface will be perpendicular to that y-axis. All cross-sections perpendicular to that y-axis will intersect the ellipsoid as an ellipse in an x–z plane. Rearranging Equation 51 gives

$$\frac{x^2}{R^2\left(1-\frac{y^2}{R^2}\right)}+\frac{z^2}{Z^2\left(1-\frac{y^2}{R^2}\right)}=1 \quad (55)$$

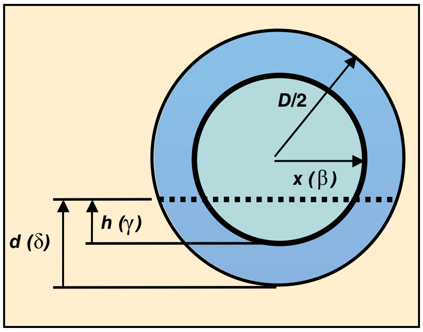

FIGURE 4. In this schematic view looking toward the left-hand dished head of a horizontal tank, the outer circle represents the cylindrical diameter and the inner circle represents a cross-section in the dished region

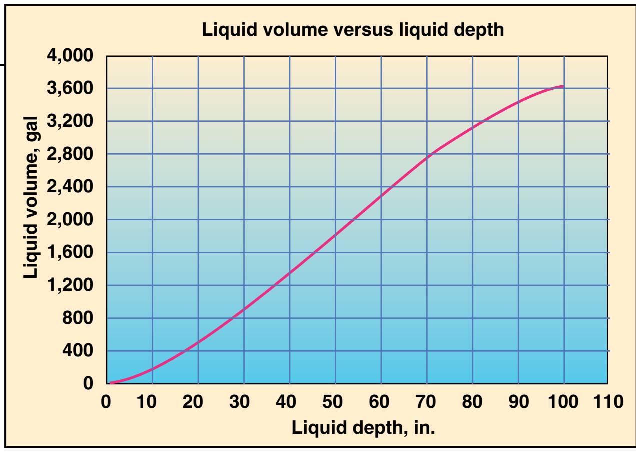

FIGURE 5. This plot shows the liquid volume versus liquid depth for an example horizontal tank

Comparing this with the general form of an ellipse in Equation 53, we see that the x-axis intercept of the ellipse in a plane perpendicular to the y-axis at any y value will be $R(1–{}^2/R^2)^{1/2}$. The corresponding z-axis intercept will be $Z(1–y^2/R^2)^{1/2}$. From Equation 54, the area of the ellipse will be

$$A = \pi RZ\left(1 - \frac{y^2}{R^2}\right) \tag{56}$$

To calculate the liquid depth in the two heads, we first recognize that the two heads combined comprise a complete ellipsoid. We calculate the liquid volume of both heads as

$$v_{LH} = \int_{-R}^{-R+d} A dy$$
$$= \pi RZ \int_{-R}^{-R+d}\left(1 - \frac{y^2}{R^2}\right)dy \tag{57}$$

Carrying out this integration and simplifying gives our final equation:

$$v_{LH} = \frac{\pi d^2}{3e}\left(\frac{3}{2}D - d\right) \tag{58}$$

For the case where $e = 1$, and heads are hemispherical, Equation 58 reduces to

$$v_{LH} = \frac{\pi d^2}{3}\left(\frac{3}{2}D - d\right) \tag{59}$$

If a tank with a hemispherical head is full ($d = D$), Equation 59 gives:

$$v_{LH} = \frac{\pi D^3}{6} \tag{60}$$

Which is the well-known formula for the volume of a sphere.

Equation 58 shows that the liquid volume (and capacity) of a true elliptical head is inversely proportional to e. Thus, for example, a true 2:1 elliptical head on a tank of a given diameter would hold exactly half the liquid volume of a hemispherical head on the same tank.

Results

The equations in this paper have been incorporated into two Microsoft Excel spreadsheets — one for vertical tanks and the other for horizontal tanks. Tables 4 and 5 show excerpts from the spreadsheet programs. (Note: Abbreviated versions of Tables 4 and 5 are shown on page here, while the full versions of both tables are available in the online version of this article at www.che.com, at the Web Extras tab.) The description of Tables 4 and 5 that follows pertains to the full website versions, but notations are made where the parts being discussed are not seen in the table excerpts that are shown in the print version here.)

The equations programmed in these spreadsheets are rather substantial. Considerable effort was expended to ensure accurate representation of the equations in the spreadsheet formulas. Readers may download the spreadsheet templates at www.che.com (Web Extras tab)..

A suggested organization would be to maintain one copy of each spreadsheet template, and then create a separate spreadsheet for each tank to which one wishes to apply the equations. So, an Excel Workbook might consist of the two spreadsheet templates, plus an individual spreadsheet for each physical tank of interest.

Below, the input parameters are identified, and the general layout of the spreadsheets is described. Then we show spreadsheet examples for a vertical tank (Table 4) and for a horizontal tank (Table 5). .For simplicity, the same tank (with different tank orientation) is used for both examples. The shaded cells are used to input the parameters for a specific vessel. The other cells are calculated by formulas.

The particular tank used in these examples has a dia. of 100-in. dia. (all dimensions are for the inside of the tank), a height or length of 120 in., and ASME F&D heads, designated as Head Style 1. The liquid has a specific gravity of 1.18.

The tank length specified in Tables 4 and 5 is the total length from end to end. It can be directly measured, allowing for the wall thickness, or determined from engineering drawings.

Some drawings may not give the overall length specifically. In these cases, the length of each head from the end to the plane where the head becomes cylindrical can be calculated by multiplying α_2 in Tables 4 or 5 by the inside tank diameter. If the drawing gives the distance between the weld beads, allowance must be made if the heads also include any cylindrical portion. If so, those lengths must be added to the length between the welds.

Four input parameters control the population of the strapping tables: (1) head style number; (2) tank diameter (in.); (3) tank length or height (in.); and (4) specific gravity of the liquid. They are entered in the top-left box in the shaded cells.

Below the input area is a box (not shown in the print version of the tables) containing head-style parameters calculated by the spreadsheet in accordance with the Head Style Number input. The values for α_1 to α_5 and f_d and f_k are supplied by formulas and are defined in the Nomenclature box.

The third box down on the left (titled Region Capacities in the print edition version of the tables) gives the calculated tank capacities for the five tank

NOMENCLATURE

Symbol	Description
a	*x*-axis intercept of an ellipse, Equation 53
A_L	Area of liquid in any cross-section perpendicular to the tank axis
b	*y*-axis intercept of an ellipse, Equation 53
C	Dimensionless proportionality constant in Equation 35
d	Liquid depth in the cylindrical region of a horizontal tank
D	Inside diameter of the cylindrical portion of tank
e	The ratio of the long to short axes of an ellipse of revolution
f_d	Dimensionless spherical radius
f_k	Dimensionless knuckle radius
h	Liquid depth at any cross-section in a dished head
H	Total inside tank height (vertical tank) or total inside tank length (horizontal tank)
R	Tank radius of cylindrical region
R_d	Radius of the spherical portion of a dished head
R_k	Radius of the knuckle curvature of a dished head
V_h	Volume of dished head
v_i	Volume of liquid in Region *i*
v_L	Liquid volume
V_i	Volume capacity of Region *i*
V_T	Volume capacity of the tank
x	Radial coordinate from the center line to the tank edge
y	Length coordinate from the bottom of the tank (vertical tank) or the left-hand end of the tank (horizontal tank)
z	*Z*-axis of an orthogonal coordinate system
Z	*Z*-axis intercept of an ellipsoid; height of a true elliptical head

Symbol	Description
α	Dimensionless height (vertical tank) or length (horizontal tank)
β	Dimensionless radius
δ	Dimensionless liquid depth in the cylindrical region of a horizontal tank
$\Delta\alpha$	Dimensionless interval in Simpson's Rule
γ	Dimensionless liquid depth at any cross-section in a dished head
θ	Angle between the tank center line and a radius drawn from the origin of the spherical radius of a torispherical head through the origin of the knuckle radius
Subscripts	
a	First cross-section in the Simpson's Rule formula
b	Second cross-section in the Simpson's Rule formula
c	Third cross-section in the Simpson's Rule formula
C	Cylindrical region of the tank
d	Spherically shaped dished head region
H	Both elliptical heads combined
k	Center point of the knuckle radius
L	Liquid
1	Plane where the bottom or left spherical region meets the adjacent knuckle region; bottom or left spherical region
2	Plane where the bottom or left knuckle region meets the cylindrical region; bottom or left knuckle region
3	Plane where the cylindrical region meets the top or right knuckle region; cylindrical region
4	Plane where the top or right knuckle region meets the adjacent spherical region; top or right knuckle region
5	Top or right-hand end of the tank; top or right spherical region

regions and for the total tank. Below that (not shown in the print version) is a lookup table that gives parameters for the various head styles. Specifying a Head Style Number in the Input Information Box pulls the appropriate values for f_d, f_k, α_1, and α_2 from this lookup table and places them in the Head Style Parameter box (not shown in the print version of these tables).

The Head Style Number, one of the required input parameters, is defined in the box just below the strapping table (again, not shown in the print version). Five choices are offered for vertical tanks and a sixth one is added for horizontal tanks. That sixth style is for a true elliptical head. If that style is chosen, then the value for the True Ellipse Ratio must be entered in the box below the strapping table. It is the ratio of the long axis to the short axis of the true-elliptical head. Most elliptical heads are fabricated using two radii of curvature to approximate the ellipse.

Different manufacturers use somewhat different radii in their approximations. The spreadsheet offers a choice between using a two-radii approximation (Style 4) or a true ellipse (Style 6). These two options might be useful if one wanted to compare how close the two radii approximation is to a true ellipse. If one had elliptical heads with f_d and f_k values other than used here for

TABLE 4. STRAPPING TABLE FOR A VERTICAL TANK

Input Information	
Tank name:	T-1000
Tank orientation	Vertical
Liquid	Aq. solvent
Head style	1
Tank dia., in.	100.0
Tank height, in.	120.0
Specific gravity	1.18

Region	
Capacities	Gal.
V_1	176.9
V_2	173.8
V_3	2,928.5
V_4	173.8
V_5	176.9
V_T	3,629.8

Depth	Liq. depth		Liq. depth	Liq. vol.	Weight
gage: %	ft	in.	in.	gal	lb
0	0	0	0	-	-
*					
10	1	0	12	188	1,850
*					
33	3	4	40	1,135	11,166
*					
100	10	0	120	3,630	35,713

* Rows not shown in this abbreviated version of this table can be found in the full version online.

Liquid Volume Calculator	
(This calculator will return the liquid volume for an input liquid level.)	
Liq. depth, in. =	12.0
α =	0.120
Liq. vol., gal =	187.99

Head Style 4, one could simply enter those values in the box below the strapping table for Head Style 4.

The large tables displayed as two panels in the upper center and upper right of Tables 4 and 5 are the strapping tables (abbreviated in the print version). There, a liquid volume and a liquid weight (calculated from the liquid density input) is shown for each 1% of the total possible liquid depth range. The 1% liquid depth increments are expressed as (1) percentages, (2) as ft and in., and (3) as in. Then each row gives the calculated liquid volume in gal and weight in lb. If one has a liquid depth that falls between two rows in the strapping table, one can interpolate. Or, a plot of the table could be constructed and used to read the volume. Or, the Liquid Volume Calculator can be used, described as follows.

At the bottom of each spreadsheet is what is called the Liquid Volume Calculator. It uses the input parameters described above along with a liquid level entered in the Liquid Volume Calculator. That value can be as exact as one cares to specify it. The Liquid Volume Calculator then calculates the

TABLE 5. STRAPPING TABLE FOR A HORIZONTAL TANK

Input Information	
Tank name	T-1001
Tank orientation	Horizontal
Liquid	Aq. solvent
Head style	1
Tank dia., in.	100.0
Tank length, in.	120.0
Specific gravity	1.18

Region	
Capacities	Gal
V_1	176.9
V_2	173.8
V_3	2,928.5
V_4	173.8
V_5	176.9
VT	3,629.8

Depth	Liq. depth		Liq. depth	Liq. vol.	Weight
gage, %	ft	in.	in.	gal	lb
0	0	0	0	-	-
*					
40	3	4	40	1,338	13,160
*					
50	4	2	50	1,815	17,856
*					
100	8	4	100	3,630	35,713

* Rows not shown in this abbreviated versiono of this table can be found in the full version online.

Liquid Volume Calculator	
(This calculator will return the liquid volume for an input liquid level.)	
Liq. depth, in. =	50.0
Liq. vol. cyl., gal =	1,464.25
Liq. vol. heads, gal =	350.65
Liq. vol., gal =	1,814.89

liquid volume for that liquid level.

Vertical tank orientation. Table 4 (abbreviated here) shows the spreadsheet output for the above-described tank oriented vertically. The total tank capacity is 3,629.8 gal, with 80.7% of that being in the cylindrical region (2,928.5 gal) and the rest being in the two heads. If one wanted to know the liquid volume for liquid depth of 40 in., for example, a table lookup would give 1,135 gal or 11,166 lb. To illustrate the Liquid Volume Calculator a liquid level of 12.0 in. was entered, which returned a liquid volume of 187.99 gal. That volume corresponds to the value in the strapping table.

Horizontal tank orientation. Table 5 (abbreviated here) displays the same tank oriented horizontally. The tank capacities of the five regions and the total tank capacity are the same as in Table 4. The difference in this spreadsheet is that the strapping table must be populated using numerical integration (Simpson's Rule) for the liquid volumes in the heads because of the complexity of the equation being integrated. That integration is performed in spreadsheet cells below those shown in Table 5 (only shown in the Excel spreadsheets available for download), with the results of the integration being carried up to the appropriate line in the Liquid Volume Calculator.

An Excel macro populates each row in the table by repeatedly carrying out the following steps: (1) copies a liquid level from the strapping table to the clipboard; (2) pastes that value into the Liquid Volume Calculator which allows the Simpson's Rule integration to be performed and the result placed in the appropriate row of the Liquid Volume Calculator; (3) copies the total liquid volume from the Liquid Volume Calculator to the clipboard;

and (4) pastes the total liquid volume from the Liquid Volume Calculator to the appropriate row of the strapping table. The macro repeats that operation for each line of the table.

After one enters these parameters for a particular tank, clicking the "Click to Run Macro to Populate" button activates the macro and populates the liquid volumes in the table. It will be necessary to enable macros in Excel if that functionality has been disabled for security reasons.

In the Simpson's Rule integration (only shown in the spreadsheets for download), the dished head is partitioned by 101 equally spaced planes perpendicular to the tank's main axis. A liquid area is calculated for each plane, and the integration is performed by Simpson's Rule to give a liquid volume for the specified depth.

So, for example, if one had a liquid height of 40 in. in the described horizontal tank, a lookup in Table 5 would give a liquid volume of 1,338 gal and a liquid weight of 13,160 lb. The Liquid Volume Calculator at the bottom of the spreadsheet works the same way as described for the vertical tank. In this example, a liquid height of 50.0 in. was entered and a corresponding liquid volume of 1,814.89 gal was returned. Figure 5 shows a plot of the liquid volume versus depth for this example. ■

Edited by Suzanne Shelley

References

1. Doolittle, A.K., Universal Tank Calibration Chart, *Ind. and Eng. Chem.*,Vol. 20, No. 3, pp. 322–323, 1928.
2. Perry, R.H. and Green, D.W.: "Perry's Chemical Engineers' Handbook," McGraw-Hill Book Co., New York, pp. 10-138–10-141, 1997.
3. Jones, Dan: Computing Fluid Tank Volumes, *Chem. Proc.*, Vol. 65 (11), November 2002, pp.. 46–50.
4. Weast, R.C., Selby, S.M., and Hodgman, C.D., "Handbook of Mathematical Tables," Chemical Rubber Co., 1964.

Authors

Daniel R. Crookston is a senior project engineer for Champion Technologies (3130 FM 521 Rd., Fresno, TX 77545; Phone: 281-431-2561; Email: daniel.crookston@champ-tech.com). He holds a B.S.Ch.E. from the University of Texas, and an M.S.Ch.E. from the University of Arkansas. Prior to joining Champion, he was employed for five years as a process/project engineer for Nalco Co. He is currently overseeing the design and construction of reaction and blending facilities in Brazil and Colombia. This is Crookston's fifth technical publication and his first in collaboration with his father.

Reid B. Crookston is a retired engineer in League City, Tex. (Email: tank.volume1@gmail.com). He holds a B.S.Ch.E. from the University of Pittsburgh, and M.S.Ch.E. and a Ph.D. in chemical engineering from Carnegie Mellon University. He worked for Gulf/Chevron for 27 years, serving as a technical consultant with Gulf R&D Co. in Pittsburgh and Houston, and overseeing Gulf's R&D work on enhanced-oil-recovery (EOR) methods. With Chevron, he was chief reservoir engineer for Tex., Mo., Okla., New Mex., and Calif. This is his seventh technical publication, and his first in collaboration with his younger son.

Input Information	
Tank name:	T-1000
Tank orientation:	Vertical
Liquid:	Aq. Solvent
Head style:	1
Tank dia., in.:	100.0
Tank height, in.:	120.0
Specific gravity:	1.18

Head Style Parameters	
α_5	1.2000000
α_4	1.0836834
α_3	1.0306624
α_2	0.1693376
α_1	0.1163166
f_d	1.000
f_k	0.060

Region Capacities	gal
V_1	176.9
V_2	173.8
V_3	2,928.5
V_4	173.8
V_5	176.9
V_T	3,629.8

Table 4. Strapping Table for a Vertical Tank

Depth Gage: %	Liq. Depth, ft	in.	Liq. Depth, in.	Liq. vol., gal	Weight, lb
0%	0	0	0	-	0
1%	0	1	1	1	13
2%	0	2	2	5	53
3%	0	4	4	21	211
4%	0	5	5	33	329
5%	0	6	6	48	472
6%	0	7	7	65	640
7%	0	8	8	85	834
8%	0	10	10	131	1,293
9%	0	11	11	159	1,560
10%	1	0	12	188	1,850
11%	1	1	13	219	2,158
12%	1	2	14	252	2,478
13%	1	4	16	319	3,138
14%	1	5	17	353	3,472
15%	1	6	18	387	3,807
16%	1	7	19	421	4,141
17%	1	8	20	455	4,476
18%	1	10	22	523	5,145
19%	1	11	23	557	5,479
20%	2	0	24	591	5,814
21%	2	1	25	625	6,148
22%	2	2	26	659	6,483
23%	2	4	28	727	7,152
24%	2	5	29	761	7,486
25%	2	6	30	795	7,821
26%	2	7	31	829	8,155
27%	2	8	32	863	8,490
28%	2	10	34	931	9,159
29%	2	11	35	965	9,493
30%	3	0	36	999	9,828
31%	3	1	37	1,033	10,162
32%	3	2	38	1,067	10,497
33%	3	4	40	1,135	11,166
34%	3	5	41	1,169	11,501
35%	3	6	42	1,203	11,835
36%	3	7	43	1,237	12,170
37%	3	8	44	1,271	12,504
38%	3	10	46	1,339	13,173
39%	3	11	47	1,373	13,508
40%	4	0	48	1,407	13,842
41%	4	1	49	1,441	14,177
42%	4	2	50	1,475	14,511
43%	4	4	52	1,543	15,180
44%	4	5	53	1,577	15,515
45%	4	6	54	1,611	15,849
46%	4	7	55	1,645	16,184
47%	4	8	56	1,679	16,518
48%	4	10	58	1,747	17,187
49%	4	11	59	1,781	17,522
50%	5	0	60	1,815	17,856

Depth Gage: %	Liq. depth, ft	in.	Liq. depth, in.	Liq. vol., gal	Weight, lb
51%	5	1	61	1,849	18,191
52%	5	2	62	1,883	18,525
53%	5	4	64	1,951	19,195
54%	5	5	65	1,985	19,529
55%	5	6	66	2,019	19,864
56%	5	7	67	2,053	20,198
57%	5	8	68	2,087	20,533
58%	5	10	70	2,155	21,202
59%	5	11	71	2,189	21,536
60%	6	0	72	2,223	21,871
61%	6	1	73	2,257	22,205
62%	6	2	74	2,291	22,540
63%	6	4	76	2,359	23,209
64%	6	5	77	2,393	23,543
65%	6	6	78	2,427	23,878
66%	6	7	79	2,461	24,212
67%	6	8	80	2,495	24,547
68%	6	10	82	2,563	25,216
69%	6	11	83	2,597	25,550
70%	7	0	84	2,631	25,885
71%	7	1	85	2,665	26,219
72%	7	2	86	2,699	26,554
73%	7	4	88	2,767	27,223
74%	7	5	89	2,801	27,558
75%	7	6	90	2,835	27,892
76%	7	7	91	2,869	28,227
77%	7	8	92	2,903	28,561
78%	7	10	94	2,971	29,230
79%	7	11	95	3,005	29,565
80%	8	0	96	3,039	29,899
81%	8	1	97	3,073	30,234
82%	8	2	98	3,107	30,568
83%	8	4	100	3,175	31,237
84%	8	5	101	3,209	31,572
85%	8	6	102	3,243	31,906
86%	8	7	103	3,277	32,241
87%	8	8	104	3,311	32,575
88%	8	10	106	3,378	33,235
89%	8	11	107	3,410	33,555
90%	9	0	108	3,442	33,863
91%	9	1	109	3,471	34,153
92%	9	2	110	3,498	34,419
93%	9	4	112	3,545	34,879
94%	9	5	113	3,565	35,073
95%	9	6	114	3,582	35,241
96%	9	7	115	3,596	35,384
97%	9	8	116	3,608	35,502
98%	9	10	118	3,624	35,660
99%	9	11	119	3,628	35,700
100%	10	0	120	3,630	35,713

Head Style	Style No.	f_d	f_k	α_1	$\alpha_2 = \alpha_k$
ASME F&D	1	1.000	0.0600	0.1163	0.1693
ASME 80/10 F&D	2	0.800	0.1000	0.1435	0.2255
ASME 80/6 F&D	3	0.800	0.0600	0.1568	0.2050
2:1 Elliptical	4	0.875	0.1700	0.1018	0.2520
Spherical	5	0.500	0.5000	0.5000	0.5000

Liquid Volume Calculator	
(This calculator will return the liquid volume for an input liquid level.)	
Liquid level, in.	12.0
α =	0.120
Liq. vol, gal =	**187.99**

Input Information	
Tank name:	T-1001
Tank orientation:	Horizontal
Liquid:	Aq. Solvent
Head style:	1
Tank dia., in.:	100.0
Tank length. in.:	120.0
Specific sravity:	1.18

Head Style Parameters	
α_5	1.2000000
α_4	1.0836834
α_3	1.0306624
α_2	0.1693376
α_1	0.1163166
f_d	1.000
f_k	0.060

Region Capacities	gal
V_1	176.9
V_2	173.8
V_3	2,928.5
V_4	173.8
V_5	176.9
V_T	3,629.8

Table 5 -- Strapping Table for a Horizontal Tank

Depth gage, %	Liq. Depth, ft	Liq. Depth, in.	Liq. Depth, in.	Liq. vol., gal	Weight, lb	Depth gage, %	Liq. Depth, ft	Liq. Depth, in.	Liq. Depth, in.	Liq. vol., gal	Weight, lb
0	0	0	0	-	0	51	4	3	51	1,863	18,330
1	0	1	1	5	51	52	4	4	52	1,911	18,804
2	0	2	2	15	146	53	4	5	53	1,959	19,277
3	0	3	3	28	271	54	4	6	54	2,007	19,749
4	0	4	4	43	420	55	4	7	55	2,055	20,220
5	0	5	5	60	589	56	4	8	56	2,103	20,690
6	0	6	6	79	777	57	4	9	57	2,151	21,159
7	0	7	7	100	982	58	4	10	58	2,198	21,626
8	0	8	8	122	1,202	59	4	11	59	2,245	22,090
9	0	9	9	146	1,436	60	5	0	60	2,292	22,553
10	0	10	10	171	1,685	61	5	1	61	2,339	23,013
11	0	11	11	198	1,946	62	5	2	62	2,385	23,470
12	1	0	12	226	2,219	63	5	3	63	2,432	23,925
13	1	1	13	255	2,504	64	5	4	64	2,477	24,376
14	1	2	14	285	2,801	65	5	5	65	2,523	24,823
15	1	3	15	316	3,107	66	5	6	66	2,568	25,267
16	1	4	16	348	3,424	67	5	7	67	2,613	25,706
17	1	5	17	381	3,751	68	5	8	68	2,657	26,142
18	1	6	18	415	4,087	69	5	9	69	2,701	26,573
19	1	7	19	450	4,432	70	5	10	70	2,744	26,998
20	1	8	20	486	4,785	71	5	11	71	2,787	27,419
21	1	9	21	523	5,146	72	6	0	72	2,829	27,835
22	1	10	22	561	5,516	73	6	1	73	2,871	28,244
23	1	11	23	599	5,893	74	6	2	74	2,912	28,648
24	2	0	24	638	6,277	75	6	3	75	2,952	29,045
25	2	1	25	678	6,667	76	6	4	76	2,992	29,436
26	2	2	26	718	7,065	77	6	5	77	3,031	29,820
27	2	3	27	759	7,469	78	6	6	78	3,069	30,197
28	2	4	28	801	7,878	79	6	7	79	3,107	30,567
29	2	5	29	843	8,294	80	6	8	80	3,143	30,928
30	2	6	30	886	8,714	81	6	9	81	3,179	31,281
31	2	7	31	929	9,140	82	6	10	82	3,214	31,626
32	2	8	32	973	9,571	83	6	11	83	3,249	31,962
33	2	9	33	1,017	10,006	84	7	0	84	3,282	32,289
34	2	10	34	1,062	10,446	85	7	1	85	3,314	32,606
35	2	11	35	1,107	10,890	86	7	2	86	3,345	32,912
36	3	0	36	1,152	11,337	87	7	3	87	3,375	33,208
37	3	1	37	1,198	11,788	88	7	4	88	3,404	33,494
38	3	2	38	1,244	12,243	89	7	5	89	3,432	33,767
39	3	3	39	1,291	12,700	90	7	6	90	3,459	34,028
40	3	4	40	1,338	13,160	91	7	7	91	3,484	34,276
41	3	5	41	1,385	13,623	92	7	8	92	3,508	34,511
42	3	6	42	1,432	14,087	93	7	9	93	3,530	34,731
43	3	7	43	1,479	14,554	94	7	10	94	3,551	34,936
44	3	8	44	1,527	15,023	95	7	11	95	3,570	35,124
45	3	9	45	1,575	15,493	96	8	0	96	3,587	35,293
46	3	10	46	1,623	15,964	97	8	1	97	3,602	35,442
47	3	11	47	1,671	16,436	98	8	2	98	3,615	35,567
48	4	0	48	1,719	16,909	99	8	3	99	3,625	35,662
49	4	1	49	1,767	17,383	100	8	4	100	3,630	35,713
50	4	2	50	1,815	17,856						

Head Style	Style No.	fd	f_k	α_1	$\alpha_2 = \alpha_k$
ASME F&D	1	1.0000	0.0600	0.116317	0.169338
ASME 80/10 F&D	2	0.8000	0.1000	0.143479	0.225544
ASME 80/6 F&D	3	0.8000	0.0600	0.156779	0.205021
2:1 Elliptical	4	0.8750	0.1700	0.101777	0.252003
Spherical	5	0.5000	0.5000	0.500000	0.500000
True Elliptical	6				
(True-Ellipse Ratio)	2				

Liquid Volume Calculator		
(This calculator will return the liquid volume for an input liquid level.)		
Liq, depth, in. =		50.0
Liq. vol. cyl., gal =		1,464.25
Liq. vol. heads, gal =		-
Liq. vol., gal =		**1,464.25**